Math 360--Homework 5:

1. Professor P is looking for help writing a computer program—“the project”. She can contract with any mix of three sources for help: undergraduate students at $8 per hour, graduate students at $10 per hour, and professional programmers at $30 per hour. The full project would take a professional programmer at least 1000 hours, but grad students are only 30% as productive, and undergraduates, 20%. The professor only has 164 hours of her own time to devote to the effort. She estimates, from experience, that undergraduate programmers will require 0.2 hours of her time for every hour of work, graduate students will require 0.1 hours of her time for every hour of work, and professional programmers will require 0.05 hours of her time for every hour of work. Her goal is to complete this project as cheaply as possible. Note: she can spend time working on the project—but then she will have less time for supervision of programmers.
2. Set up and solve this linear program. Write your solution and optimal value below.
3. Professor P doesn’t like programming—how does the solution and value change if she can only supervise and not program?
   1. Given this new information, what is Professor P’s hourly “wage”? Hint: How much more does the project cost AND how many less hours is she now working?
4. Continued from (b): Her school has a graduate research assistant with work study for one semester (600 hours). If she is being required to use and pay this student, how much does this change the solution and value?

**Sensitivity (from c)**:

1. How much would the hourly rate of graduate student programmers need to be reduced for Professor P to hire out more hours?
2. How much would the hourly rate of undergraduate student programmers need to be reduced for Professor P to hire out more hours?
3. How much would the hourly rate of professional programmers need to increase for Professor P to reduce the number of professional hours?
4. One of Professor P’s colleagues has expressed interest in helping as a way to earn additional income. At what price per hour should Professor P be interested if she estimates the colleague would be 80% as efficient as a professional and require 0.1 hour of supervision per hour of work?
5. Tube Steel Incorporated (TSI) is optimizing production at its four mills. TSI makes eight types of tubular products which are either solid or hollow and come in four diameters. The following two tables show production costs (in dollars) per tube of each product at each mill and the extrusion times (**in minutes**) for each allowed combination. Missing values indicate product-mill combinations that are not feasible. (You can copy/paste these into excel)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Unit Cost ($)** | | | |
| **Product** | Mill 1 | Mill 2 | Mill 3 | Mill 4 |
| 0.5 in. solid | 0.1 | 0.1 |  | 0.15 |
| 1 in. solid | 0.15 | 0.18 |  | 0.2 |
| 2 in. solid | 0.25 | 0.15 |  | 0.3 |
| 4 in. solid | 0.55 | 0.5 |  |  |
| 0.5 in. hollow |  | 0.2 | 0.13 | 0.25 |
| 1 in. hollow |  | 0.3 | 0.18 | 0.35 |
| 2 in. hollow |  | 0.5 | 0.28 | 0.55 |
| 4 in. hollow |  | 1.0 | 0.6 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Unit Time (in minutes)** | | | |
| **Product** | Mill 1 | Mill 2 | Mill 3 | Mill 4 |
| 0.5 in. solid | 0.5 | 0.5 |  | 0.1 |
| 1 in. solid | 0.6 | 0.6 |  | 0.3 |
| 2 in. solid | 0.8 | 0.6 |  | 0.6 |
| 4 in. solid | 0.1 | 1.0 |  |  |
| 0.5 in. hollow |  | 1.0 | 0.5 | 0.5 |
| 1 in. hollow |  | 1.2 | 0.6 | 0.6 |
| 2 in. hollow |  | 1.6 | 0.8 | 0.8 |
| 4 in. hollow |  | 2.0 | 1.0 |  |

Yearly minimum requirements for the solid sizes (in thousands) are 250, 150, 150, and 80, respectively. For the hollow sizes they are 190, 190, 160, and 150 (again in thousands). Each mill can run one, two, or three 40-**hour** shifts a week, 50 weeks a year. In other words, each mill can operate 40, 80, or 120 **hours** per week, 50 weeks a year. That’s 40 hours/week \* 50 weeks \* 60 minutes = 120,000 minutes per year, up to a maximum of 360,000 minutes per year. Note: mills don’t have to work exactly 120, 240, or 360…anything between 120 and 360 is fine.

1. Set up and solve this linear program.
2. Use your sensitivity analysis to list the shadow prices for each of the eight products. In business, these are referred to as marginal costs. This is the actual cost for producing each of these products. For example, see below:

We needed to produce 190,000 1-inch hollow tubes. The shadow price is $0.29, so it would cost $0.29 to produce one more 1-inch hollow tube. This cost would stay constant up to producing 67,500 more 1-inch hollow tubes. Note: these tubes could be produced in mill 2, 3, or 4. But mill 2 takes twice as long to produce a 1-inch tube than either mills 3 or 4. Thus Solver picked mills 3 and 4 to produce 1-inch tubes and that choice collectively made the price $0.29.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Constraints | |  |  |  |  |  |  |
|  |  |  | **Final** | **Shadow** | **Constraint** | **Allowable** | **Allowable** |
|  | **Cell** | **Name** | **Value** | **Price** | **R.H. Side** | **Increase** | **Decrease** |
|  | $F$20 | 1 in. hollow | 190 | 0.29 | 190 | 67.5 | 11.66666667 |

1. How much could costs be reduced by closing a mill? (this could be answered with a sensitivity analysis—or you could add a constraint to close a mill and see how the price changes.) How much could be saved? [Note: there would be additional savings in payroll, electricity, rent, and insurance from closing a mill—you are just stating the manufacturing savings.]
2. Continuing from (c), management is considering opening mills on weekends, which would add up to 16 extra hours per shift (up to 48 more **hours per week**), how would this new policy effect the optimal value? [Note: there would be additional costs for the extra shifts—but let’s neglect these—tell management how much they could save, then they can figure how much to spend on the extra shifts.]
3. Restarting from (b): Instead of closing mill 4, another option management is considering is to hire an industrial engineer to find ways to reduce unit costs of production at high-cost mill 4. For each of the six products produced at mill 4, taken separately, use sensitivity results to determine to what level unit costs would have to be reduced before there could be any change in the optimal production plan.
4. A final option management is considering for mill 4 is to install equipment to produce 4-inch solid and 4-inch hollow tubes. The new equipment would produce either product in 1 minute per unit. Taking each product separately, determine the unit production cost that would have to be achieved to make it economical to use the new facilities.